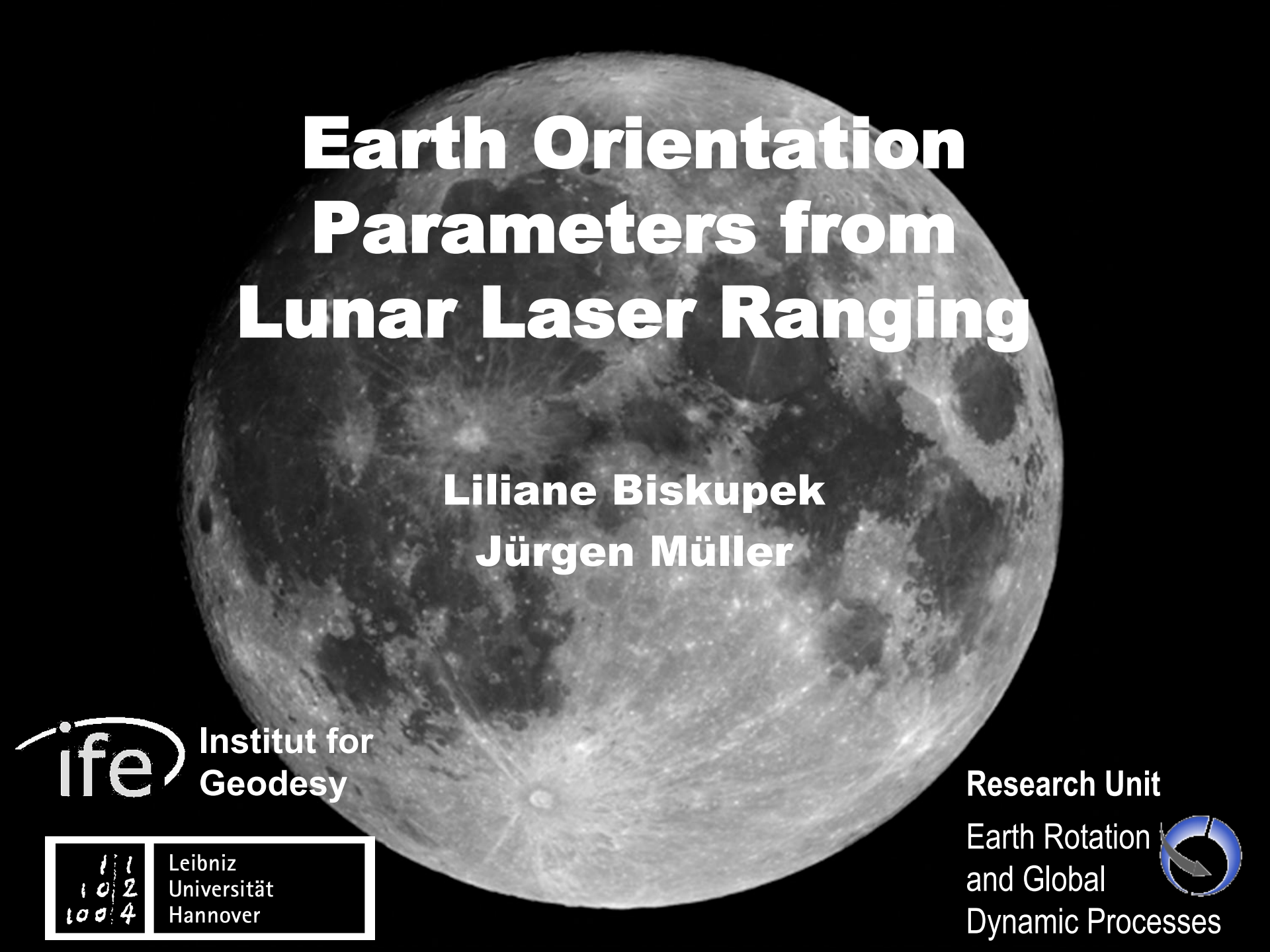

Earth Orientation Parameters from Lunar Laser Ranging

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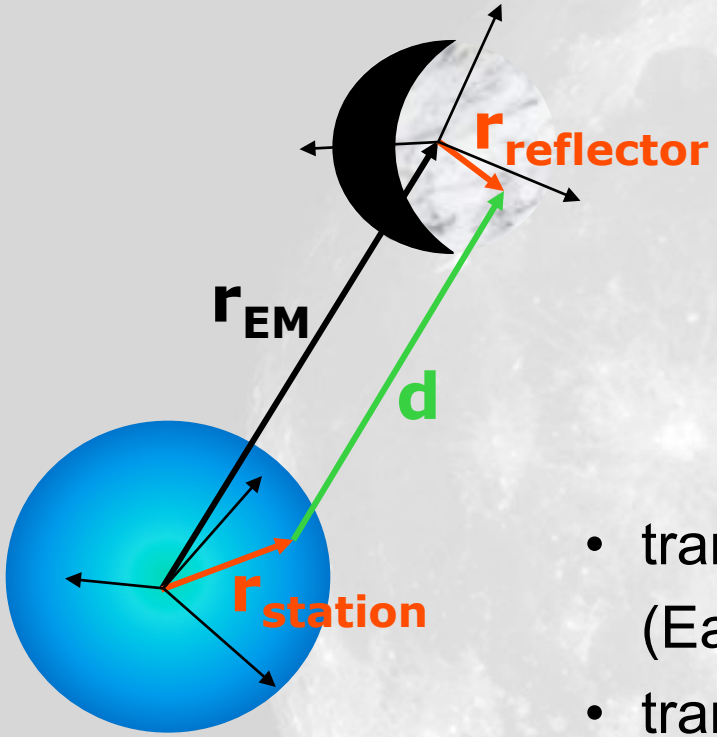
Research Unit

Earth Rotation and Global Dynamic Processes

Contents

- Model and analysis
- Nutation
- Earth rotation from LLR data by daily decomposition method
- Conclusions

Model



- model based upon Einstein's theory

$$d = c\frac{\tau}{2} = \left| \mathbf{r}^i_{EM} - \mathbf{r}^i_{station} + \mathbf{r}^i_{reflector} \right| + c\Delta\tau$$

$$\frac{d^2\mathbf{r}^i_{EM}}{dt^2} = -\frac{GM_{E+M}}{r^3_{EM}}\mathbf{r}^i_{EM} + \mathbf{b}_{Newtonian} + \mathbf{b}_{Relativity}$$

- transformation between reference systems (Earth, Moon, inertial)
- transformation between time systems
- orbital motion of the solar system bodies
- rotation of Earth and Moon
- gravitational time delay (Shapiro effect)

$$\mathbf{r}^i_{station} = \mathbf{R}(t) \cdot \mathbf{r}^{ITRF}_{station}$$

$$\mathbf{r}^i_{reflector} = \mathbf{R}(t) \cdot \mathbf{r}^{SRF}_{reflector}$$

Analysis

- weighted least-squares adjustment:
  → determination of the parameter of Earth-Moon system (ca. 180)
  - coordinates of LLR stations and retro-reflector arrays
  - parameters of physical librations and orbit of the Moon
  - orbit / mass of the Earth-Moon system,
  - lowest mass multipole moments of the Moon
  - long-periodic nutation parameters
  - relativistic parameters

Analysis

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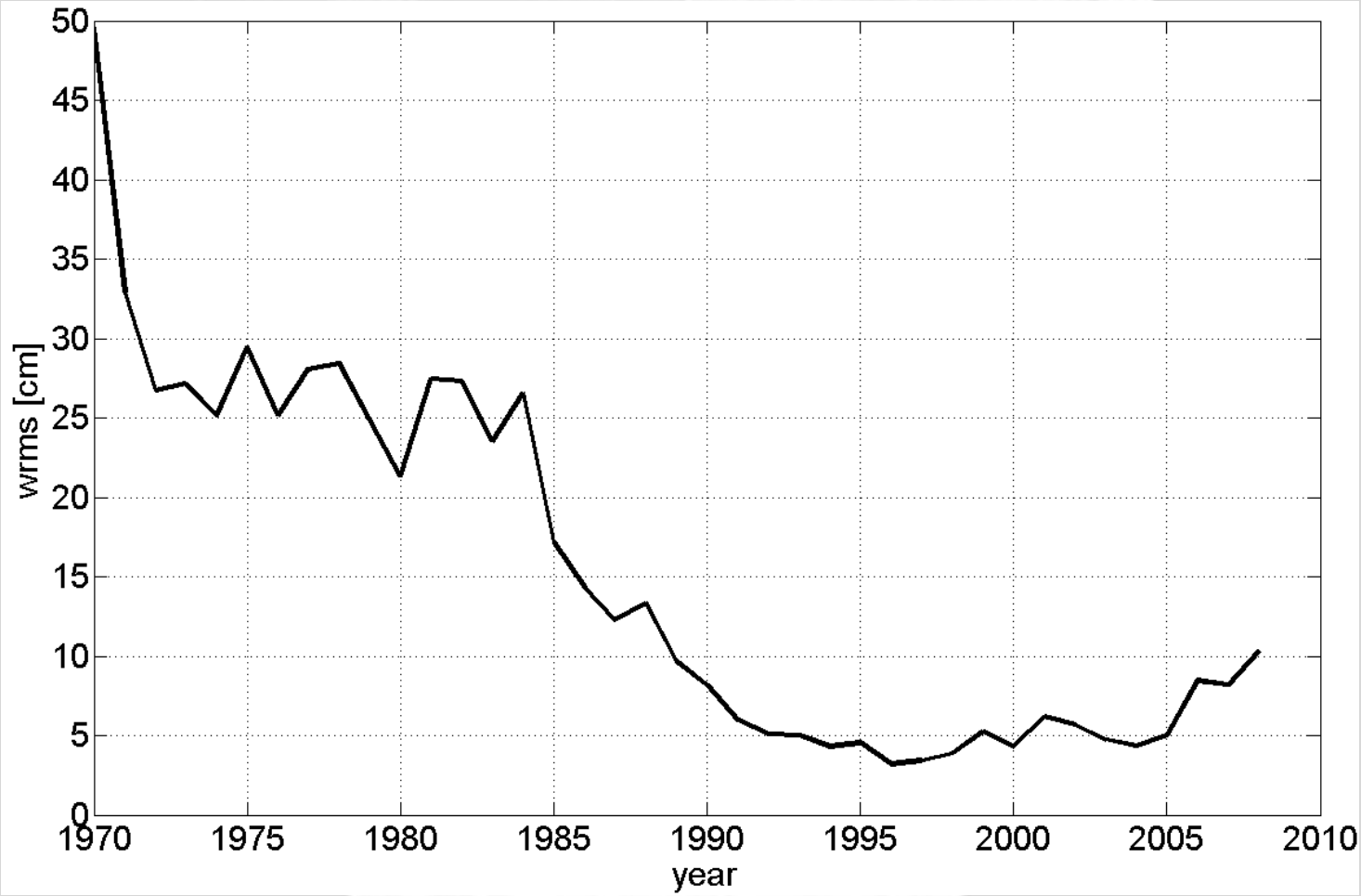
  → residuals of standard solution

  - long-periodic nutation parameters
  - relativistic parameters

Standard solution 2008

- weighted residuals of data 12.1969 - 03.2008 (16230 normal points)



Nutation

- Fitting for different period of nutation coefficients
  - 18.6 years, 9.3 years, 1 year, 182.6 days,13.6 days

$$\Delta\psi = \sum_{i=1}^{N} \left(A_i + A_i' t\right)\sin\left(ARG\right) + \left(A_i'' + A_i''' t\right)\cos\left(ARG\right)$$

$$\Delta\varepsilon = \sum_{i=1}^{N} \left(B_i + B_i' t\right)\cos\left(ARG\right) + \left(B_i'' + B_i''' t\right)\sin\left(ARG\right)$$

$$ARG = \sum_{j}^{5} N_j F_j$$

N_j : multiplier, F_j : Delaunay parameter

Nutation

MHB2000 model	A_i [mas]	B_i [mas]	A_i'' [mas]	B_i'' [mas]
18.6 year	-17206.42	9205.23	3.34	1.54
182.6 days	-1317.09	573.03	-1.37	-0.46
13.6 days	-227.64	97.85	0.28	0.14
9.3 year	207.46	-89.75	-0.07	-0.03
1 year	147.59	7.39	1.18	-0.19

own results	A_i [mas]	B_i [mas]	A_i'' [mas]	B_i'' [mas]
18.6 year	-17201.93	9203.41	3.84	3.88
182.6 days	-1316.88	572.98	-3.25	-0.98
13.6 days	-230.54	99.26	0.16	0.31
9.3 year	207.13	-90.75	1.63	-0.21
1 year	146.83	7.86	0.27	-0.58

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Earth rotation from LLR data

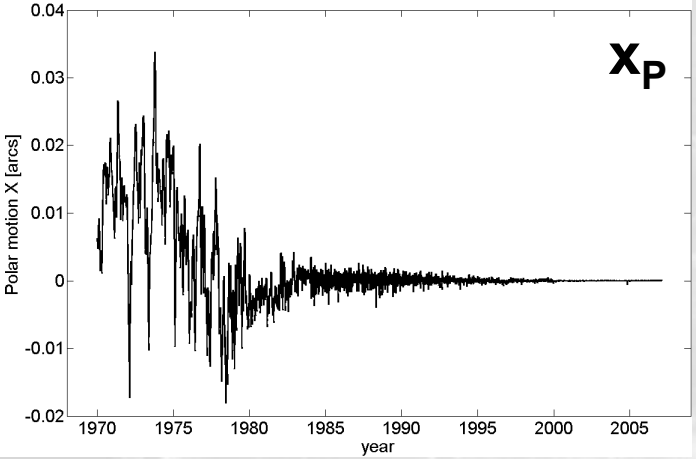
- use of different EOP series (IERS EOP C04, COMB2006) as input in global adjustment
- analysis of the post-fit residuals to determine corrections for Earth rotation ΔUT0 and variation of latitude VOL
- use of determined VOL corrections in own analysis to improve the results

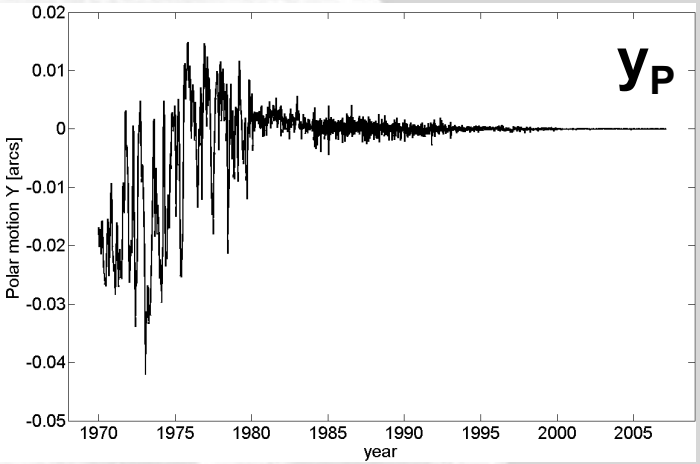
Data sets of EOP (1)

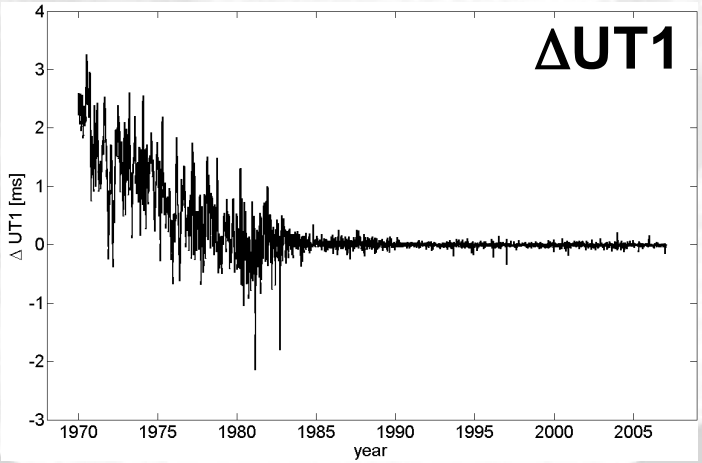
	IERS EOP C04	COMB2006
data	VLBI GPS SLR LLR optical observation	VLBI GPS SLR LLR optical observation
tidal effects	5 days - 18.6 years Defraigne and Smith (1999)	5 days - 35 days Yoder et al. (1981)

Data sets of EOP (2)

- differences between IERS EOP C04 and COMB2006







Daily decomposition (1)

- merge of post-fit residuals to daily sets of station-reflector combinations (min. 3)
  - 1179 daily sets for OCA, Grasse
  - 752 daily sets for Mc Donald
  - daily-decomposition method (least-squares adjustment)

$$r(t) = r_{\Delta\phi} + r_{\Delta UT0} + r_n$$

Daily decomposition (2)

$$r_{\Delta\phi} = 2\,\Delta\phi\,a_E\,(\sin\phi\cos\delta\cos H - \sin\delta\cos\phi)$$

$$r_{\Delta UT0} = 2\,\Delta UT0\,a_E\cos\phi\sin H\cos\delta$$

$$\Delta\phi = x_P\cos\lambda - y_P\sin\lambda$$

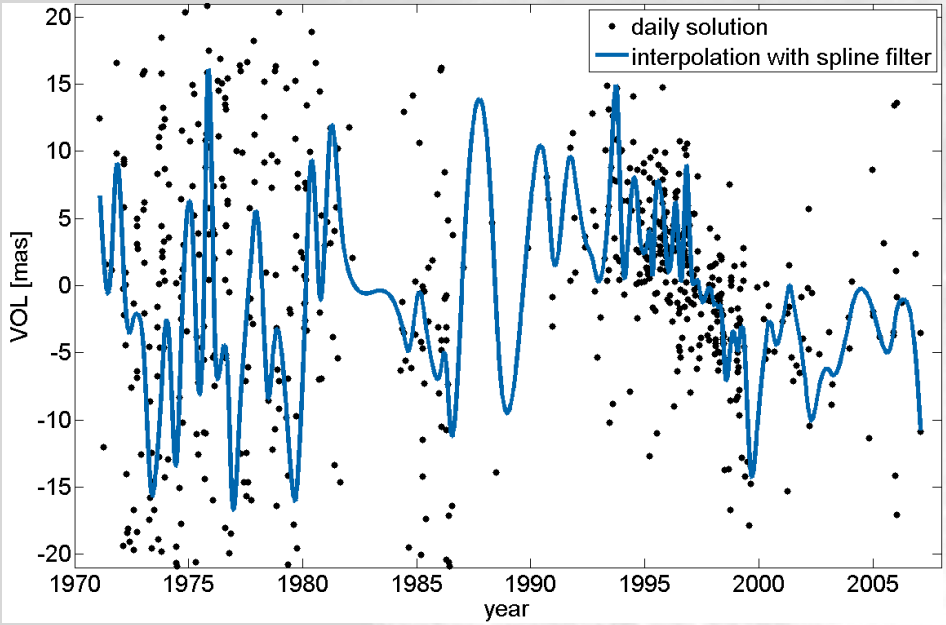
$$\Delta UT0 = \Delta UT1 + \tan\phi(x_P\sin\lambda + y_P\cos\lambda)$$

δ : declination
H : hour angle
ϕ : latitude
a_E : distance to rotation axis

Results for VOL

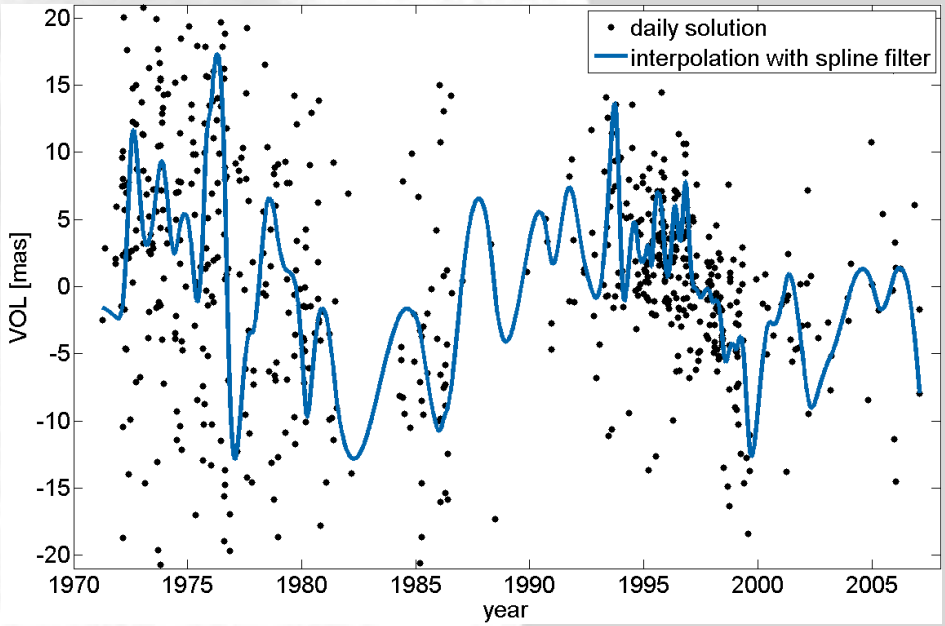
- Mc Donald, Texas

IERS EOP C04



std = 6.1 mas

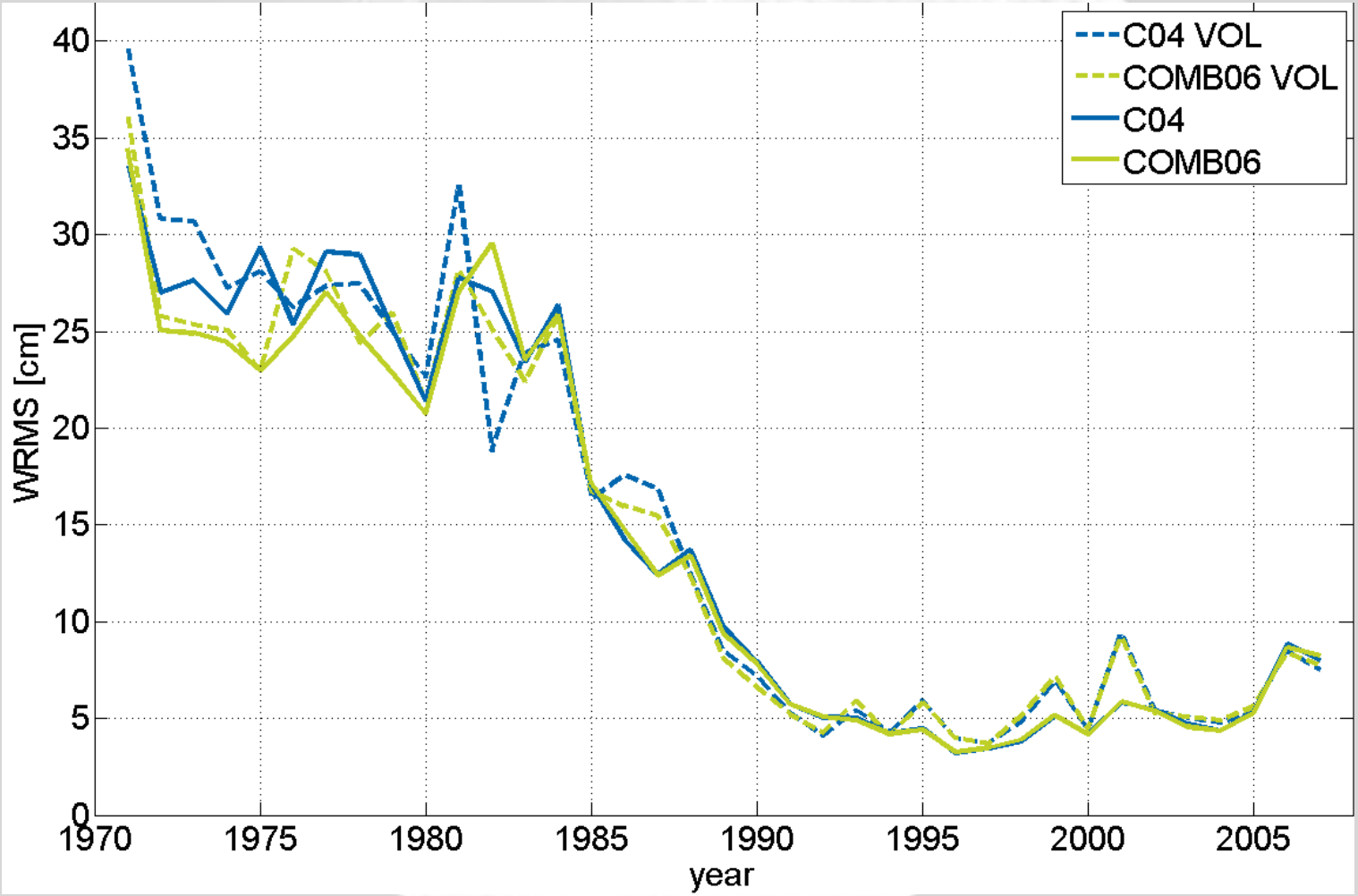
COMB2006



std = 6.4 mas

Corrections in global adjustment

- use of VOL as correction in calculation



Conclusions

- Long-periodic nutation coefficients
  - differences to the model not yet understood, further investigation needed
  - comparison with VLBI results

- Corrections for ΔUT0 and VOL are calculated
  - use of VOL in own analysis does not show significant improvements in the results
  - test of different filters
  - calculation of LOD to compare with VLBI results